②

χ'_-: $c_1 = \frac{\cos\theta - 1}{\sin\theta} e^{-i\phi} c_2$

$= \frac{-\cancel{2}\sin^2\theta/2}{\cancel{2}\cancel{\sin(\theta/2)}\cos(\theta/2)} e^{-i\phi} c_2$

$= -\frac{\sin\theta/2}{\cos\theta/2} e^{-i\phi} c_2$

$c_1 = \sin\theta/2 \, e^{-i\phi/2}$

$c_2 = -\cos\theta/2 \, e^{+i\phi/2}$

$$\chi'_+ = \begin{pmatrix} \cos(\theta/2) e^{-i\phi/2} \\ \sin(\theta/2) e^{+i\phi/2} \end{pmatrix} \qquad \chi'_- = \begin{pmatrix} \sin(\theta/2) e^{-i\phi/2} \\ -\cos(\theta/2) e^{+i\phi/2} \end{pmatrix}$$

Check: $\theta \to \pi - \theta$, $\phi \to \phi + \pi$ effectively exchanges χ'_+ and χ'_-

$\chi'_+ \cdot \chi'_- = \chi'^*_{+1} \chi'_{-1} + \chi'^*_{+2} \chi'_{-2}$

$= \cos\theta/2 \, \cancel{e^{i\phi/2}} \sin(\theta/2) \cancel{e^{-i\phi/2}} + \sin(\theta/2) \cancel{e^{-i\phi/2}} [-\cos\theta/2] \cancel{e^{+i\phi/2}}$

$= 0$ ✓